WAVE MOTION

Definitions

Amplitude: The maximum displacement of any particle from its mean position is called amplitude of the wave

Period: The time taken for any particle to complete one vibration is called period of the wave

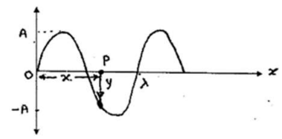
Frequency: The number of vibration per second by a particle is called frequency (n). $n = 1/T$

Wavelength: The distance between two consecutive particles of the medium which are in same phase

Velocity: The distance travelled by the wave in one second is called velocity of the wave. $v = \lambda/T = n\lambda$

Define Simple Harmonic Progressive Wave and derive its equation in different forms:

Wave which travels continuously in a given direction and the particles of the medium perform SHM about their mean position is called simple harmonic progressive wave.



Let us consider a simple harmonic progressive wave travelling in the +ve X-axis direction. If y represents its displacement and x represents the position of the particle in the medium, then the relation between them can be graphically shown.

At instant t, the displacement of the particle at origin (x=0) is given by

$$y = A \sin(\omega t)$$

where A is the amplitude and ω the angular frequency.

Let us consider a particle P at a distance x from the origin O. This particle lags behind the origin particle by some angle θ, because the disturbance reaches P after some time. Therefore, the displacement of P is given by

$$y = A \sin(\omega t - \theta)$$

Since two particles differing by position λ, differ in phase by 2π, therefore for P which is x distance away from origin, the phase difference is

$$\theta = \frac{2\pi x}{\lambda}$$

Therefore, $$y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

$$y = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right), since\ \omega = \frac{2\pi}{T}$$

$$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$y = A\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$$

$$y = A\sin 2\pi n\left(t - \frac{x}{n\lambda}\right)$$

$$y = A\sin 2\pi n\left(t - \frac{x}{v}\right), since\ v = n\lambda$$

$$y = A\sin 2\pi n\left(\frac{vt - x}{v}\right)$$

$$y = A\sin 2\pi\left(\frac{vt - x}{\lambda}\right)$$

NOTE: If the wave travels in the negative X direction, then all the above equations will have + instead of –

What are beats and explain the formation of beats and derive the expression for beat frequency:

When two sound waves having the same amplitude and slightly different frequencies arrive at a point simultaneously they produce interference. The resultant intensity of sound at that point varies periodically with time from maximum to minimum. When sound intensity becomes maximum, it is called waxing and minimum intensity is called waning. This phenomenon of waxing and waning of sound is called phenomenon of beats.

One waxing and the next waning is called one beat and the time interval between two successive waxings or wanings is called beat period. The number of beats produced per second is called beat frequency.

Consider two sound waves having amplitude A and frequencies n1 and n2 respectively such that $|n1 - n2|$ is small.

$y_1 = A \sin 2\pi n_1 t$ and $y_2 = A \sin 2\pi n_2 t$

By principle of superposition, $y = y_1 + y_2$

$y = A \sin(2\pi n_1 t) + A \sin(2\pi n_2 t)$

$$y = 2A\sin\left(2\pi\left(\frac{n_1+n_2}{2}\right)t\right)\cos\left(2\pi\left(\frac{n_1-n_2}{2}\right)t\right)$$

Let $\frac{n1+n2}{2} = n$= mean frequency and $2A\cos\left(2\pi\left(\frac{n1-n2}{2}\right)t\right) = R$

Where R=resultant Amplitude

Thus, $y = R \sin(2\pi nt)$

This shows that the resultant is SHM of frequency n, mean of the frequencies, and amplitude R, which is variable

Case 1: Waxing: R = ±2A, when $\cos\left(2\pi\left(\frac{n1-n2}{2}\right)t\right) = \pm 1$

thus $2\pi\left(\frac{n1-n2}{2}\right)t = 0, \pi, 2\pi \ldots. p\pi$

$$Thus, t = \frac{p}{n_1 - n_2}, where\ p = 0,1,2,3, \ldots\ldots$$

$$Thus, we\ get\ waxing\ at\ t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, \ldots\ldots. and\ so\ on$$

$$Thus, the\ time\ between\ two\ consecutive\ waxings\ is\ \frac{1}{n_1 - n_2}$$

Case 2: Waning: R = 0, when $\cos\left(2\pi\left(\frac{n1-n2}{2}\right)t\right) = 0$

thus $2\pi\left(\frac{n1-n2}{2}\right)t = \frac{\pi}{2}, \frac{3\pi}{2} \ldots. \frac{(2p+1)\pi}{2}$

$$Thus, t = \frac{2p+1}{2(n_1 - n_2)}, where\ p = 0,1,2,3, \ldots\ldots$$

$$Thus, we\ get\ waning\ at\ t = \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \ldots\ldots. and\ so\ on$$

$$Thus, the\ time\ between\ two\ consecutive\ wanings\ is\ \frac{1}{n_1 - n_2}$$

Thus, the time between two successive waxing and waning is same

Beat period = $\frac{1}{n_1 - n_2}$ $and\ beat\ frequency = n_1 - n_2$

Application of Beats:

>>Determination of unknown frequency

The sound note of unknown frequency is sounded simultaneously with a note of known frequency which can be adjusted. The known frequency is so adjusted that beats are heard. The further adjustment is made till beats reduce to zero. The two frequencies are now equal.

>>The phenomenon of beats can be used to produce low frequency notes used in Jazz orchestra or western music

>>Musical instruments can be tuned by noting beats produced when two different instruments when sounded together. By adjusting the frequency of one of the instruments, the number of beats is reduced to zero. The two instruments are now emitting notes of same frequency. The instruments are now in unison with each other.

Doppler Effect in Sound:

The apparent change in frequency of sound for a listener whenever there is relative motion between the source and the listener is called Doppler effect in sound.

Case 1: Source is stationary and observer is moving

>> If the observer is moving towards the source then

$$n_a = \left(\frac{V + Vo}{V}\right) n$$

Thus, the apparent frequency increases

>> If the observer is moving away from the source then

$$n_a = \left(\frac{V - Vo}{V}\right) n$$

Thus, the apparent frequency decreases

Case 2: Observer is stationary and source is moving

>> If the source is moving towards the observer then

$$n_a = \left(\frac{V}{V - Vs}\right) n$$

Thus, the apparent frequency increases

>> If the source is moving away from the observer then

$$n_a = \left(\frac{V}{V + Vs}\right) n$$

Thus, the apparent frequency decreases

NOTE: If there is no relative motion between the observer and source (or both are stationary) then $n_a = n$

Applications of Doppler's Effect:

>>The traffic police use speed guns which are fixed for a certain speed. If the vehicle passing by passes at a higher speed, then beats are produced and an alarm is initiated. Thus Doppler effect is used for speed detection on highways.

>>RADAR: It emits continuous high frequency electromagnetic waves called radio waves. These waves on hitting the object (like an airplane) will get reflected and mix (super position) with the constant signal to produce beats. From this beat frequency the speed of the object can be determined.

>>It can also be used to determine the speed of the star. If the star is moving towards the earth, the spectral lines are more towards violet end of the spectrum of if the star is moving away then the spectral lines are more towards the red end. This is called as Doppler shift and helps to determine the speed of the star.

>>In colour Doppler sonography, the ultrasonic waves is refracted from the body tissue and can give information about the rate of flow of various fluids including blood.

Limitation:

>>The velocity of the source of sound and the observer should be much less than the velocity of sound

>>The motion of the observer and source must be along the same straight line

>>The medium such as air, in which the observer and the source are situated is at rest.

Explain change of phase of reflected waves

A. Transverse Waves:

>>Reflection from rigid medium: On reflection from denser medium (like a wall), crest is reflected as a trough and vice versa. Particle and wave velocity is reversed. There is a phase change of 180º or π radians

>>**Reflection from rarer medium:** On reflection from a rarer medium, crest is reflected as a crest and trough as a trough. Wave velocity is reversed. Particle velocity no change. No change of phase.

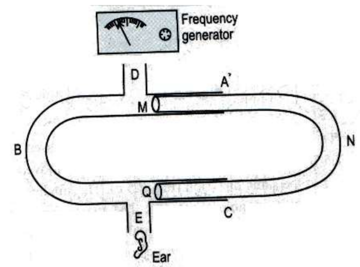
B. Longitudinal Waves:

>>Reflection from rigid medium: On reflection from denser medium (like a wall), compression is reflected as a compression. Particle and wave velocity is reversed. There is a phase change of 180º or π radians

>>**Reflection from rarer medium:** On reflection from a rarer medium, compression is reflected as a rarefaction. Particle velocity and wave velocity are in opposite directions. No change of phase.

Principle of Superposition and Constructive and Destructive Interference using Quincke's Tube:

Principle of superposition state: When two or more waves travelling through a medium arrive at a point simultaneously, each wave produces its own displacement at that point independent of the other. Hence the resulting displacement at that point is the vector sum of all the displacements due to all the waves.



The sound generated by the frequency generator travels through DEB and DNE and super imposes at E. If the path difference between the above two paths is 0, λ, 2λ,….. nλ then constructive interference takes place at E and we get a sound of maximum intensity at E. If the path difference is λ/2, 3λ/2,…… (2n+1)λ/2 then destructive interference takes place at E and we get sound of minimum(zero) intensity at E.